

## ABSTRACT

### A CONTINUOUS SQUARE ROOT INFORMATION FILTER-SMOOTHER WITH DISCRETE DATA UPDATE

J. K. Miller

Member of the Technical Staff

Jet Propulsion Laboratory

California Institute of Technology

4800 Oak Grove Drive, MS301/125J

Pasadena, CA 91109

A data filter processes data in order to obtain an estimate of parameters that are related to the data by a mathematical model. Data filters exist in many forms and use the covariance of the State parameters, or some equivalent representation, along with the measurements and a simulation of the measurements including partial derivatives, to obtain the desired estimate. The covariance matrix of the state may be represented by its inverse or information matrix or square root factorizations of either of these matrices.

Data filters may be separated into two categories being continuous filters or discrete filters. Continuous data filters are described by a matrix differential equation or Ricatti equation and discrete filters are obtained by solving the continuous equations over some finite time interval. Discrete data filters have been developed to a high degree of efficiency and are used almost exclusively for processing data.

Continuous data filters have an advantage of inherent simplicity. The complete elimination of the need to compute a state transition matrix results in a simple algorithm involving numerical integration. The continuous filter thus promises increased accuracy since the error control may be placed directly on the covariance, or representation of the covariance, rather than on the state transition matrix.

A continuous matrix differential equation for the SRIF filter has been derived and a computer algorithm developed to implement this filter. The filter algorithm performs the mapping of state and process noise by numerical integration of the SRIF matrix and admits data via a discrete least square update. A smoothing algorithm is also described.

Accuracy, computational efficiency, memory requirements and simplicity of design are compared with other filter algorithms. This comparison consisted of counting the number of equations that must be integrated and the number and dimension of arrays. A preliminary evaluation of these competing criteria reveals no significant differences. However, the continuous SRIF filter is expected to outperform discrete filters with regard to accuracy and memory requirements because there is no need to compute a state transition matrix and error control may be placed directly on the elements of the SRIF matrix. Simplicity of design seems to favor the continuous SRIF filter particularly when the same numerical integration algorithm used for the state propagation is used to integrate the SRIF matrix.

The integration of process noise makes the continuous SRIF filter somewhat computationally inefficient. However, an exact result is obtained and discrete filters often approximate the integration of process noise by assuming that the stochastic parameters are constant over the update time interval. When process noise is admitted to the continuous filter as discrete updates, the computational efficiency is comparable to the discrete filter implementation.

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